

The Physics of the Air Suspension Wheel

Wheel Rolling Assisted by Planetary Hub Action

By Zoltan A. Kemeny, PhD

Summary

The Air Suspension Wheel (ASW), has a rigid rim and a multiplicity of air-shocks, which are eccentric to the hub and thus are capable to transfer torque between the hub and the rim.

The towing and driving a rigid wheel on level and inclined road is revisited for introduction and comparison, followed by the same for ASW of the same weight, load and size.

In that comparison, depending on ASW size and load, the kinetic energy of a ASW is increased by 17%. The ASW is thus slower to start and to stop, but it is easier to jump over common road obstacles, without affecting the hub elevation. Since in typical vehicles the wheels represents only a fraction of the total weight, this difference in driving may not be pronounced.

The elastic hub-to-rim displacement of an ASW makes the axel load downward eccentric due to the axel load and forward eccentric due to towing or driving forces, which assist the towing or driving (planetary actions). That assistance is a function of the spoke stiffness and is expressed in reduced rolling resistance, measured in percentage of the axel load. Replacing a wheel with ASW can reduce rolling friction by 30%. Depending on the vehicle's fuel consumption, that may transpire in 4-15% maximum and 3-12% average fuel savings and emission and pollution reduction. The lower is the mileage the higher is the savings. Softer ASWs save more than harder ones. ASWs save four times more fuel in mining trucks than in racing cars.

While driving-assistance is inverse rolling-resistance and thus it is mathematically identical to free rolling on downgrade, it is insufficient for locomotion, because it requires external power by maintained towing or driving.

ASWs are best used on off-road vehicles in mining, construction, agriculture forestry and in the military. Aircraft landing gear wheels and drag racers we suggested and studied, but not field tested yet. Bullet proof security service AWSs and ASW run flat inserts were also studied extensively.



Introduction

The ASW resembles a spokes wheel with the spokes replaced by non-radial air cylinders. We shall call the shock-absorbing air cylinders shocks. Figure 1 illustrates a ASW with twelve eccentric shocks of the adiabatic compressed-gas type, thus making the ASW elasticity commensurate with that of an inflated tire. The gravity load on the hub, which is the tributary vehicle weight, uniformly compresses the shocks just turned under the hub and decompresses the ones above. This effect is not temporary. The shocks are pressurized with dry air or nitrogen, pushing the pistonhead with equal force at midstroke to push or pull, as forced upon wheel rolling.

Towing forward, uniformly compresses the shocks turned just in front of the hub and decompresses the ones behind, offsetting the hub forward. Towing backward is the same in reverse.

The torque from driving forward uniformly compresses the shocks in the far side and decompresses the ones in front. Driving in reverse does the same in reverse. These two effects are temporary.

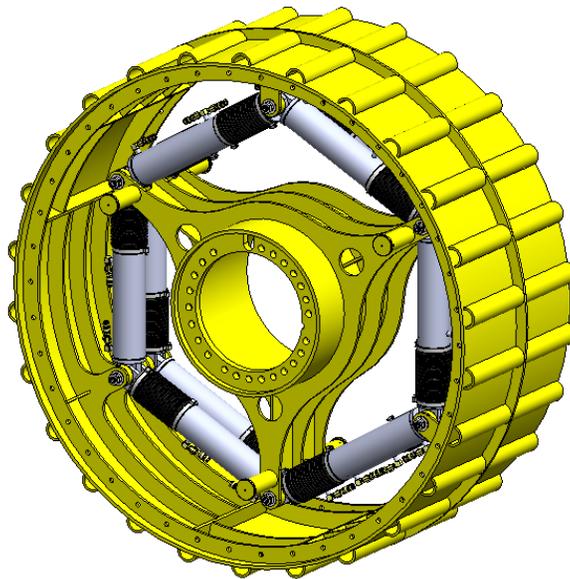


Fig. 1 Illustration of an off-road ASW without lateral constraint bullet shield

To facilitate understanding the physics of the ASW rolling, it will be instructive to revisit the rolling of rigid wheels on flat and inclined rigid roads, while being towed, driven or left alone. Then, in comparison, the same studies presented for the ASW will allow to examine the benefits and consequences of substituting wheels running on inflated tires with ASWs. We will show that such substitution results in rolling resistance reduction and in considerable fuel savings thereof. Limiting the scope to physics, we will not discuss here other details, benefits and workings of the ASW.

Notice that in Figure 1, the ribs indent into the road ensuring friction at rolling (rolling friction), and by that, ensuring a rack pinion locking kinematic constraint between the road and the ASW. That means that a towed wheel is resisted from sliding on the road by a backward pointing contact friction force and a driven wheel is resisted from turning by a forward pointing contact friction force, both resulting in wheel rolling. Thus, a rigid wheel's towing by pulling its axis and driving by engine torque around its axis, may be considered equivalent by substituting the torque by a rimpull force, equal to the torque divided by the wheel radius ($F=T/r$), acting at the contact, opposing the contact friction force (gripping). Remember that a vehicle's non-driven wheels are either pushed or pulled by the driven ones, so both pushing and pulling are considered equivalent towing forces. For illustration, see Fig. 2, where the pinion represents the wheel and the rack the road. The gear and teeth dramatize contact friction here. All computational vehicle dynamics programs assume rack-and-pinion wheel-to-road kinematic constraint.

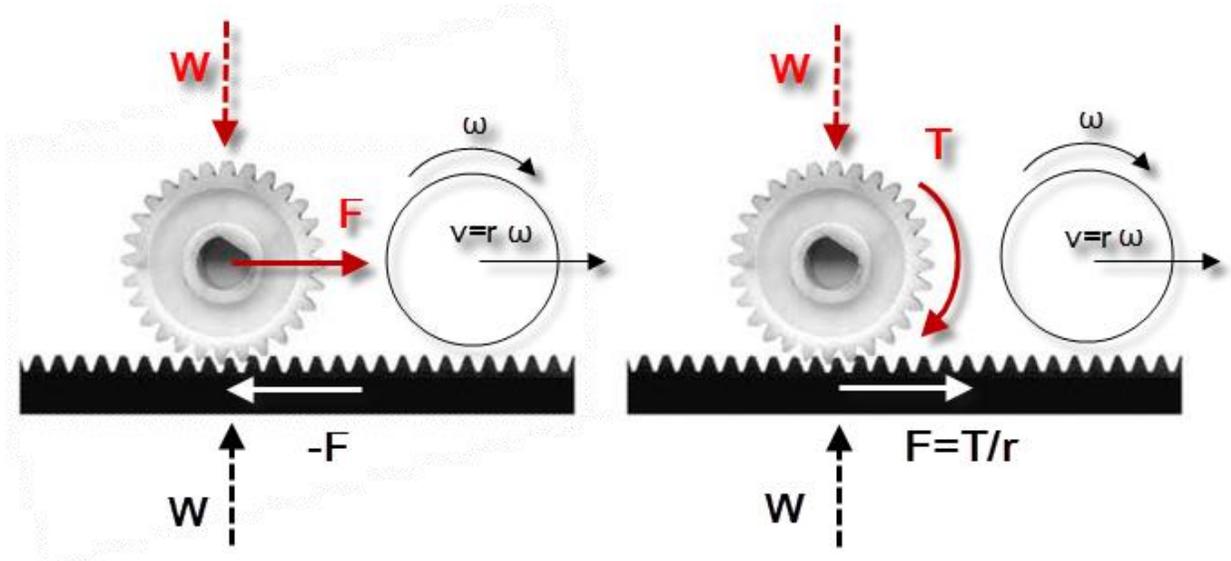


Fig. 2 Friction prevents sliding at left and rotating at right, both resulting in forward rolling.

Observe that axle load W passes through the wheel center and thus it does not cause rotation. W is gravity load, so $W=mg$, where m is the mass carried by the wheel and g is the gravity acceleration ($g=32.2$ ft/s). Towing force F on the hub is opposed by friction force $-F$, and thus, for being prevented from sliding, under the influence of the resulting torque $T=rF$, the wheel turns rolling (shown on the left). Driving torque T around the axis is opposed by friction force $F=T/r$, and thus, for being prevented from rotating in place, the wheel moves rolling (shown on the right). Hence the equality of the driving and towing, for both requiring the same forces, torques and power ($P=vF$), alas with pointing differently, yet causing the same rolling. These result in the same translational velocity v and angular velocity ω , where $v=r\omega$ and $\omega=2\pi f$, where f is the wheel rotational frequency ($f=1/T_1$, where T_1 is the time of one wheel revolution). In both cases, the wheel is accelerated by $a=F/m$.

If brake-on prevents the towed wheel from turning in snow, it will push forward and pile up snow in front of it. If winch-tieback prevents the driven wheel from moving forward, it will throw and pile up snow behind it. The same goes with sand, dirt and mud. In the rack-pinion, if the gear were stronger, the rack tooth in front of the engagement (Fig. 1 left) and behind (Fig. 1 right) the gear would brake correspondingly. If the rack were stronger however, the gear tooth behind (Fig. 1 left) and in front (Fig. 1 right) would brake correspondingly. This explains some tire and road damages. Visualizing the road as treadmill, one may see that these same forces act, whether the wheel drives the treadmill (road) or vice versa.

The rolling Motion of a Rigid Wheel

The wheel rolling motion is the result of the rotational and translational motions superposition constrained by the absence of contact slipping ($v=r\omega$), skidding ($v<r\omega$), and spinning ($\omega<v/r$), all three prevented by the contact friction. Friction is defined by $\mu_F=F/N$, where μ_F is the sliding friction coefficient, the ratio of the friction force to the normal force at sliding. It opposes the force, which accelerates the wheel. Staring the rolling from rest, the wheel turning is opposed by the static friction μ_S . That is 10-50% larger than μ_F . After start, μ_S quickly drops to μ_F . That in turn, quickly reduces to μ , which is the rolling resistance, which finally maintains rolling. At braking the wheel, this sequence reverses and sliding friction first and static friction second, will finally stop the wheel in its rolling. That is, μ_S starts and μ maintains the rolling. At $v \rightarrow 0$, $\mu \rightarrow \mu_S$, thus μ_S also ends the rolling motion. The rolling resistance is also called kinetic friction. Just like static friction, it can also be expressed in angular therm. The static friction is defined as the angle of the steepest slope, on which an object would not slide down by itself just yet.

The circumferential velocity superposition of rolling is illustrated in Fig. 3.

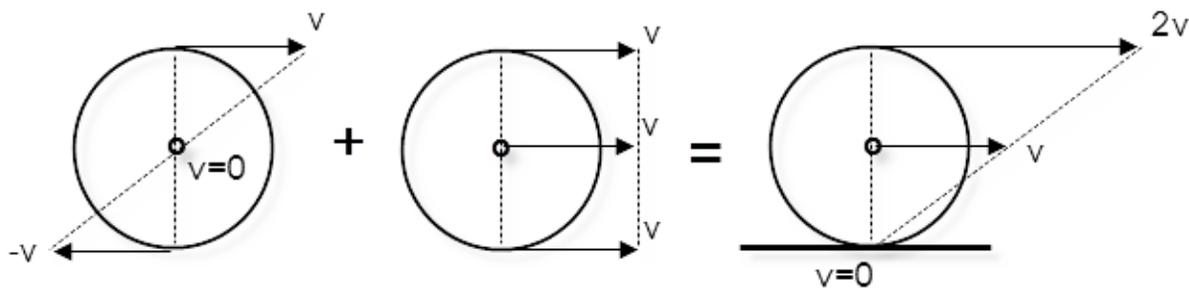


Fig. 3 The velocities cancels at the contact and doubles opposite to it (cyan vector)

The velocity of the most and least advancing points on the perimeter remains v, pointing down and up respectively (not shown). The circumferential points follow a cycloid trajectory.

The kinetic energy stored in an unloaded rolling wheel is the sum of the kinetic energies stored in that wheel in rotation and in translation. That is, $E_K = mv^2/2 + mI\omega^2/2 = mv^2[1 + (K/r)^2]/2$, where $(K/r)^2$ is 1 for rings (hoops), $3/4$ for ASWs, $1/2$ for discs (cylinders and rods) and $2/5$ for solid balls. For vehicles, having the wheels to total weight ratio is small, $(K/r)^2 \rightarrow 0$ is a good approximation. Here, I is the mass inertia around the wheel axis, k is a geometrical coefficient related to it, and m is the wheel mass. Wheels with inflated tires may be assumed to be discs, while with spokes (including ASWs), to be rings. A ASW accordingly stores 17% more kinetic energy than a wheel of the same size and weight with inflated tire. The difference of the kinetic energy of a car or truck with inflated tires or ASWs however is only 2-3%.

Rolling wheels are analyzed using free body diagrams in which forces are analyzed as if the rigid body were not rotating at all, while torques as if it were not translating at all.

Based on the towing and driving equivalency, next, only the wheels in towing will be analyzed. Five cases will be presented in didactical sequence, including free rolling wheel on slope, towing rigid wheels, towing ASWs, free rolling ASW on slope, and finally, towing ASW on slope. All wheels will be assumed vehicular wheels. The weight of such wheels are only a small fraction of the total loaded weight of the vehicle ($W_w \ll W_v$, where W_w is the wheels weight and W_v is the vehicle weight, and furthermore, $W_1 \ll W_T$, where W_1 is the weight of one wheel and W_T is the vehicle's partial weight, tributary to that one wheel). The wheel load will be denoted by W and will be assumed gravity load.

Free Rolling Rigid Wheel on Slope

Figure 4 illustrates the forces acting on a rigid vehicular wheel rolling down on a rigid road on a slope of angle ϑ under $W = mg$ gravity load on its hub.

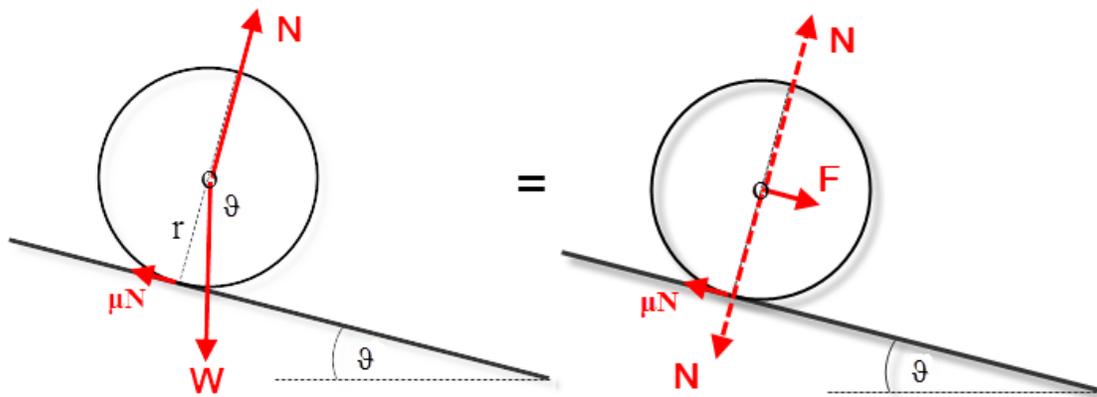


Fig. 4 Force $F - \mu N$ accelerates the wheel downhill

Three forces act upon the wheel: 1) Gravity is pulling down vertically by W on the hub, 2) the slope is pushing up by normal force N , and 3) friction, opposing down sliding, by friction force μN . The normal force is opposed and cancelled by the normal component of W as $N=W\cos\vartheta$. The parallel component of W , $F=W\sin\vartheta$, is opposed by μN . The resulting force $(F-\mu N)$ accelerates the wheel down the slope by $a= g(\sin\vartheta-\mu\cos\vartheta)$. From this we obtain the instantaneous wheel velocity as $v=at =tg(\sin\vartheta-\mu\cos\vartheta)=r\omega$ and the distance s traveled as $s=vt=at^2/2=t^2g(\sin\vartheta-\mu\cos\vartheta)/2$. Thus, the time to attain velocity v is $t=v/[g(\sin\vartheta-\mu\cos\vartheta)]$ and the time to cover travel distance s on the slope is $t=\sqrt{2sg(\sin\vartheta-\mu\cos\vartheta)}$.

Notice that the wheel is restrained from sliding down, so it continuously pivots around instantaneous contact points. Also that, N has no component in the direction of the slope, so it does not contribute to the wheel's downward acceleration.

By neglecting the mass of the wheel, we may obtain the kinematic energy of this wheel as $E_K=mv^2/2=Wv^2/(2g)=Wt^2g(\sin\vartheta-\mu\cos\vartheta)/2=sW$.

Note that in the common case of small angles, $\sin\vartheta=\tan\vartheta =\vartheta$ (measured in radians) and $\cos\vartheta\approx 1$, so the accelerating force simplifies to $F=W(\vartheta-\mu)$.

Towing Rigid Wheels

Figure 5 illustrates the dynamics of towing the same wheel on level road ($\varphi=0$).

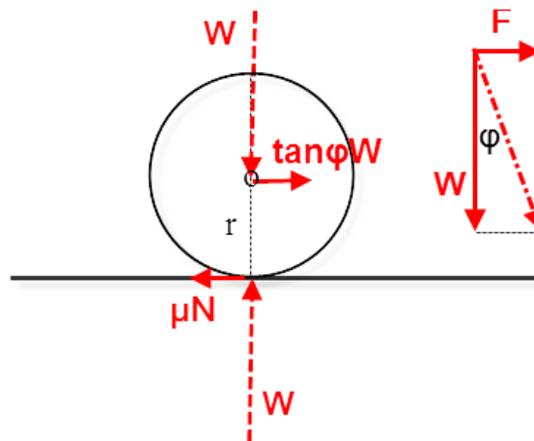


Fig. 5 Force $F-\mu N$ accelerates the wheel ($F=\tan\varphi W$)

Towing force $F=\tan\varphi W$ is now considered to be the horizontal component of an inclined hub load of inclination angle φ . The weight on the hub passes through the wheel center and cancelled by the normal force $N=W$, so it does not rotate the wheel. F would be needed to hold the wheel steady on a slope of inclination φ . This makes the case of towing, equivalent to the case of free rolling on

slope. Thus, similarly we find the three forces acting on the wheel as 1) W , 2) $N=W$ and 3) μN correspondingly. Accelerating force $F=W(\tan\phi-\mu)$ will now accelerate the wheel by $a=g(\tan\phi-\mu)$. From this we obtain the instantaneous wheel velocity as $v=at=gt(\tan\phi-\mu)=r\omega$ and the distance s traveled as $s=vt=at^2/2=t^2g(\tan\phi-\mu)/2$. Thus, the time to attain velocity v is $t=v/[g(\tan\phi-\mu)]$ and the time to cover travel distance s on the slope is $t=\sqrt{2sg(\tan\phi-\mu)}$.

Similarly, we derive the kinematic energy as $E_K=mv^2/2=Wv^2/(2g)=Wt^2g(\tan\phi-\mu)/2=sW$.

Using the small angle rules stated above, substituting $\tan\phi-\mu$ with $\sin\phi-\mu$ would make this case more akin to the previous one. That would also simplify this accelerating force to $F=W(\phi-\mu)$. One would need this much towing force to hold this well on a slope of $\vartheta=\phi$.

Towing ASWs

Figure 6 illustrates the dynamics of towing the same wheel on level road, however converted to ASW. Such ASW holds its hub in its rim by central symmetrical elasticity of shocks stiffness k . Consequently, its hub deflects downward by y under push of W and forward by x under the pull of F . The towing-force to wheel-load ratio is ϕ and the equivalent slope angle is ϑ .

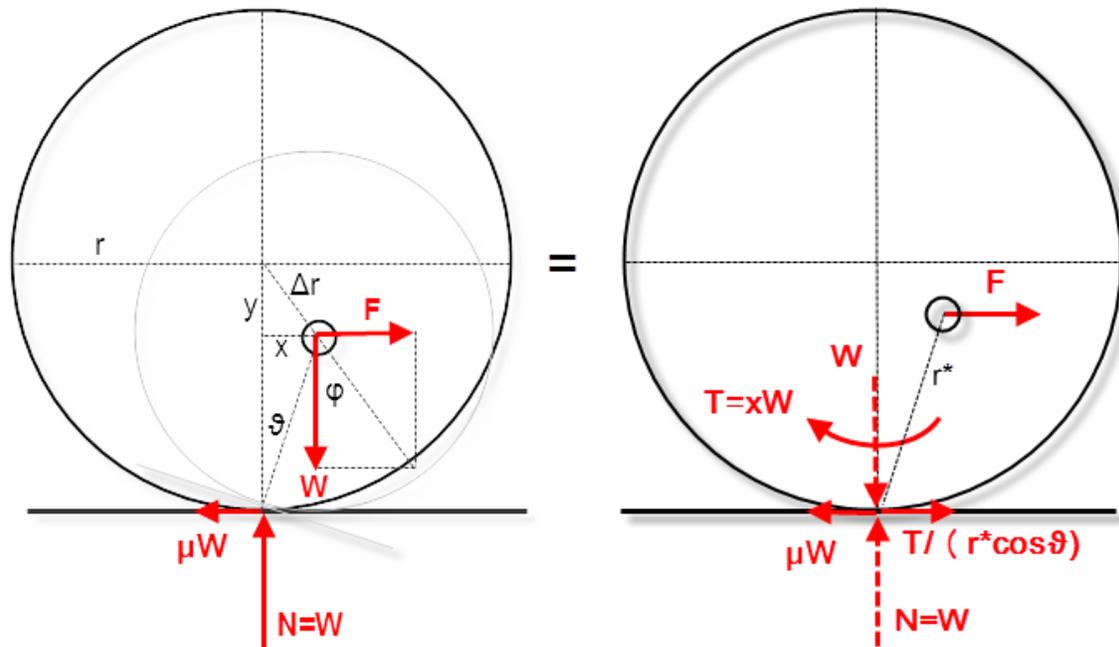


Fig 6. Forces acting upon as ASW in horizontal towing

In comparison to the previous two cases, notice that while $N=W$ now passes through the rim center, it has a lever arm (horizontal hub eccentricity x) to the hub. Similarly, gravity load W on the hub has the same horizontal eccentricity x to the pivot point, which is the rim-to-road contact point.

We have resolved the eccentric gravity load into a concentric normal force $N=W$ and a torque $T=xW$, which is then balanced by rimpull force $T/(r^*\cos\vartheta)$, which opposes the friction force μW . Now, the sum of the horizontal forces $[F-\mu W+T/(r^*\cos\vartheta)]=[F-\mu W+xW/(r^*\cos\vartheta)]$ will accelerate the ASW. This can be simplified as $F+W[x/(r^*\cos\vartheta)-\mu]=[F+W(\tan\vartheta-\mu)]=[F+W(\mu^*-\mu)]$, where x and r^* is a function stiffness k and μ^* , the rolling assistance, which is a function of $\varepsilon=W/(kr)$, the shock-spoke contraction ratio, as $\mu^*=\varepsilon\tan\varphi/(1-\varepsilon\tan^2\varphi)$. Considering small angles again, we get $\mu^*=\varepsilon\varphi$ so the accelerating force will be $F+W(\varepsilon\varphi-\mu)$. That is, an added $\varepsilon\varphi W$ force will amplify the towing force by the first order. Upon the second order, which incorporates the influence of the elastic deformation on the force (the so called P- Δ effect), the added towing force will be $\varepsilon(1+\varepsilon^{1+\varepsilon})\varphi W$, where $\varepsilon(1+\varepsilon^{1+\varepsilon})\varphi=\rho$.

The ASW appears to be equivalent to a smaller, r^* , radius wheel on slope ϑ . Yet, it is obvious that it rolls on its rim of radius r . The conjugate wheel of radius r^* is illustrated by a phantom circle. A phantom line shows the equivalent slope $\vartheta=\rho$. As a towing assistance, the forward acceleration of slope ϑ adds to the forward acceleration of towing force F . Hence comes the economy of the wheel-to-ASW replacement. For towing and free rolling on slope, the amplification factor over acceleration is then $(\varphi-\mu+\rho)/(\varphi-\mu)=[\varphi-\mu+\varepsilon(1+\varepsilon\varphi)]/(\varphi-\mu)$. That means that the ASW amplifies towing angle φ by a factor of $1+\varepsilon(1+\varepsilon^{1+\varepsilon})$. At $\varepsilon_{MAX}=\pi/9$, that factor will be 1.433. Thus, towing the substitute ASW requires up to $100(1-1/1.433)=30\%$ less towing force and power than the wheel, which was substituted. The same applies, when the towing force is substituted with driving torque and caps the possible fuel savings thereof.

Having smaller rolling radius and larger contact radius, makes the ASW unique, with attributes beneficial to rolling and towing. Smaller rolling radius r^* , lowers the vehicular center of mass by y , resulting in increased driving stability. The larger than r^* contact radius r , reduces rolling resistance and thus reduces engine power need. It saves fuel thereof.

Accelerating force $F=W(\rho+\varphi-\mu)$ will now accelerate the wheel by $a=g(\rho+\varphi-\mu)$. From this we obtain the instantaneous wheel velocity as $v=at =tg(\rho+\varphi-\mu)=r\omega$ and the distance s traveled as $s=vt=at^2/2=t^2g(\rho+\varphi-\mu)/2$. Thus, the time to attain velocity v is $t=v/[g(\rho+\varphi-\mu)]$ and the time to cover travel distance s on the slope is $t=\sqrt{2sg(\rho+\varphi-\mu)}$.

Similarly, we derive the kinematic energy as $E_K=mv^2/2=Wv^2/(2g)=Wt^2g(\rho+\varphi-\mu)/2=sW$.

Note that air cylinders of compression ratio of 2, used for shocks, are nonlinear, hardening as $\varepsilon=W/(kr^\gamma)$, where $1.2<\gamma<1.6$ for adiabatic compression cycles. For dry air, the heat capacity ratio is $\gamma=7/5$. Thus, ρ is conservative. Shocks of higher compression ratio designed with $\varepsilon=W/hr^2$, which makes ρ even more conservative. Some shock are filled with monoatomic gas of high γ .

Free Rolling ASW on Slope

Figure 7 illustrates the dynamics of free rolling of the same ASW down the slope ϑ .

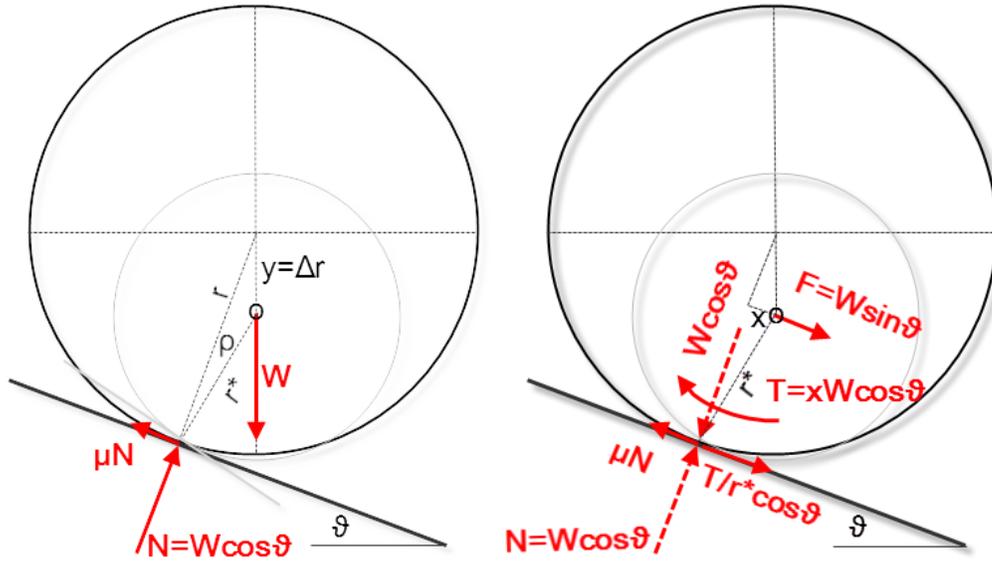


Fig. 7 Forces acting upon a ASW in free rolling on a slope

Following the same procedure, resolving torque $T = xW\cos\vartheta = W\epsilon r/\tan\vartheta$, we find that the ASW is now accelerated by force $F\sin\vartheta - \mu W\cos\vartheta + Wx\cos\vartheta/r^*\cos\vartheta$, where the normal force eccentricity is $x = y/\sin\vartheta = \epsilon r/\sin\vartheta = r^*\sin\rho = r(1 - \epsilon\cos\vartheta)\sin\rho/\sin\vartheta$, where the rolling radius is $r^* = r(1 - \epsilon\cos\vartheta)/\sin\vartheta$ and the strain is $\epsilon = W/(kr)$. After substitutions, assuming small angles and accounting for the P- Δ effect, we get the accelerating force $F = W(\rho + \vartheta - \mu)$, where the rolling assistance, expressed as angle $\rho = \epsilon(1 + \epsilon^{1+\epsilon})\vartheta$.

Accelerating force $F = W(\rho + \vartheta - \mu)$ will now accelerate the wheel by $a = g(\rho + \vartheta - \mu)$. From this we obtain the instantaneous wheel velocity as $v = at = tg(\rho + \vartheta - \mu) = r\omega$ and the distance s traveled as $s = vt = at^2/2 = t^2g(\rho + \vartheta - \mu)/2$. Thus, the time to attain velocity v is $t = v/[g(\rho + \vartheta - \mu)]$ and the time to cover travel distance s on the slope is $t = \sqrt{2sg(\rho + \vartheta - \mu)}$.

Similarly, we derive the kinematic energy as $E_k = mv^2/2 = Wv^2/(2g) = Wt^2g(\rho + \vartheta - \mu)/2 = sW$.

Note that $F\sin\vartheta$ upward pointing force would hold this ASW on this slope motionless. The hub would then deflect toward the slope orthogonally by $\epsilon R\cos\vartheta$. For not rolling, the forces remain static, no dynamics would involve, and the ASW would behave a wheel, except for its elasticity normal to the slope. That would be a singular case of this and the previous free rolling case.

Towing ASW on Slope

Fig. 8 illustrates two towing cases. On the left, towing our studied ASW downhill, and on the right, towing uphill on slope ϑ .

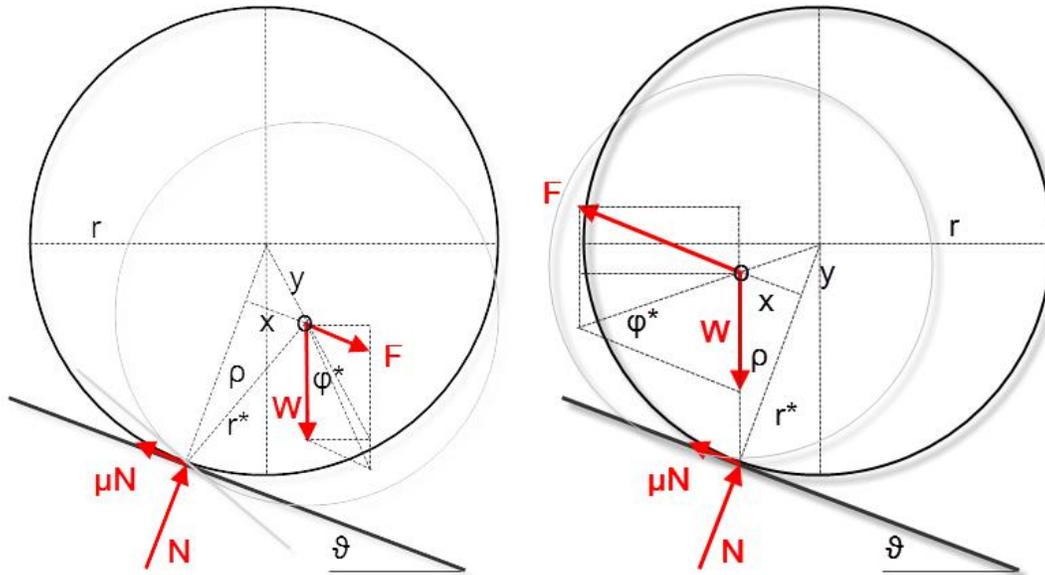


Fig. 8 Forces on the downhill (left) and uphill (right) towed ASW

Since the towing force F is always parallel with the slope, now $\varphi^*=F/(W\cos\vartheta)$, otherwise all other notations hold. Without further resolving for torque and components parallel and orthogonal to the slope, we can observe that weak towing force accelerates downward fast, while strong towing force accelerates uphill slow. We may notice that, while at uphill towing, as that shown on the right, even when W passes through the contact point, it has a large component orthogonal to slope ϑ , so it has a torque, which accelerates upward by torque. However, it also has a smaller component, which does the opposite. Thus, unless the hub is ahead of the contact point, there is no benefit from the shock-spoke deflections.

Omitting the involved derivations, we find that the accelerating force $F=W(\rho+\varphi+\vartheta-\mu)$ will now accelerate the wheel by $a=g(\rho+\varphi+\vartheta-\mu)$ downward. Upward $F=W(\rho+\varphi-\vartheta-\mu)$ will accelerate by $a=g(\rho+\varphi-\vartheta-\mu)$. From this we obtain the instantaneous wheel velocity downhill as $v=at=tg(\rho+\varphi+\vartheta-\mu)=r\omega$ and the distance s traveled as $s=vt=at^2/2=t^2g(\rho+\varphi+\vartheta-\mu)/2$. Uphill $v=at=tg(\rho+\varphi-\vartheta-\mu)=r\omega$ and $s=vt=at^2/2=t^2g(\rho+\varphi-\vartheta-\mu)/2$.

Thus, the time to attain downhill velocity v is $t=v/[g(\rho+\varphi+\vartheta-\mu)]$ and the time to cover travel distance s on the slope is $t=\sqrt{2sg(\rho+\varphi+\vartheta-\mu)}$. Uphill $t=v/[g(\rho+\varphi-\vartheta-\mu)]$ and $t=v/[g(\rho+\varphi-\vartheta-\mu)]$.

Similarly, we derive the downhill kinematic energy as $E_K=mv^2/2=Wv^2/(2g)=Wt^2g(\rho+\varphi+\vartheta-\mu)/2=sW$ and uphill as $E_K=mv^2/2=Wv^2/(2g)=Wt^2g(\rho+\varphi-\vartheta-\mu)/2=sW$

Rolling Resistance

Fig. 9 illustrates the angular interpretation of the sliding friction and the rolling resistance. It is practical to express all coefficient is such a consistent term, for being visual, it may help in understanding rolling under such a wide range of conditions as we just have studied here.

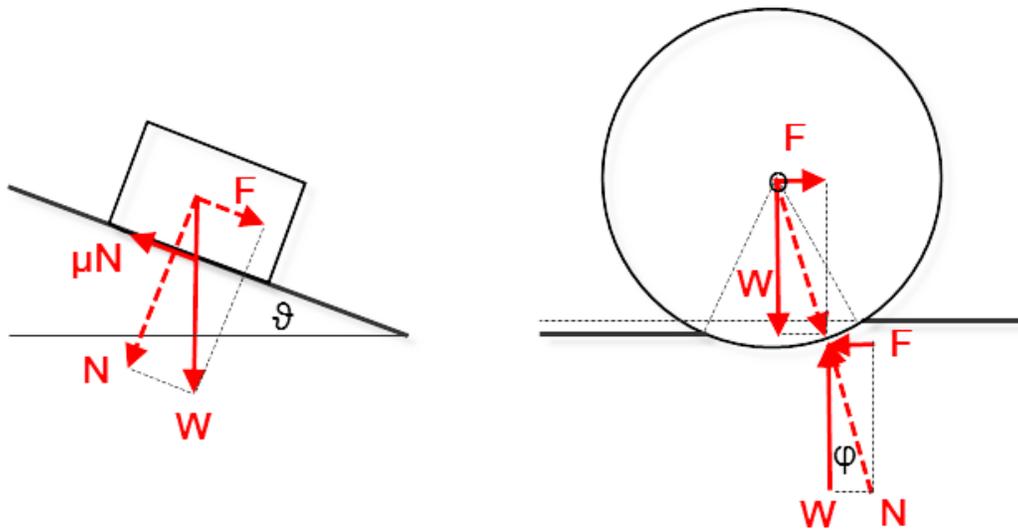


Fig. 9 Comparison of sliding and rolling resistance

We have revisited friction and rolling resistance above. Here we only give a visual comparison. Both of these has a higher initial value at static equilibrium and a lower value, which remains constant during the consecutive sliding or rolling respectively. When $\theta = \mu$ on the left and $\phi = \mu$ on the right, then we can consider sliding and rolling friction respectively.

Note that, the rolling friction, is a function of the wheel penetration, which is large on soft road and small on hard road. The literature is abundant in discussions on this and offers numerous relationships and values for these resistances.

Physical rolling resistances are based on the $\phi = \sqrt{z/2r}$ relationship, where z is the penetration in the road (sinkage depth). Reference 3 Gives other formulae used for railroads and list typical rolling resistance values in various fields of application.

In construction, mining, agriculture and military, empirical values are used. For instance, $\phi = 0.02 + 0.015z$, where z measured in inches [4]. Theoretical values overestimate measured values on soft roads, such as silt, clay, sand, soil, mud and their combination. The softer is the road, the less accurately $\sqrt{z/2r}$ predicts ϕ . The rolling resistance is often expressed in lbs towing force per tons of vehicle load term or in equivalent road grade in percentage (20 lbs/ton=1% grade). Some literature defines rolling resistance in terms of penetration depth (e.g., 2 mm).

Comparison

We summarize some benefits through the physics of the rigid wheel and the ASW for comparison in Table 1, where the angular assumptions are $\vartheta < \pi/12$, $\mu < \pi/10$ and $\varphi < \pi/8$, all measured in radians. Again, ϑ denotes road actual slope angle, μ denotes rolling resistance, ρ rolling assistance, and φ denotes towing angle, defined as the towing force to gravity axel-load ratio, measurable as an angle in the force-composition vector-diagram. To ensure non-slipping rolling, the rolling resistance must be smaller than the sliding friction coefficient, which is also an angle, the angle of a slope on which the wheel, with its brakes on, would not slide down just yet. The literature is abundant in listings of sliding and rolling resistance coefficients. Some useful reference values are listed in Ref. 1. The rolling assistance is defined as $\rho = \varepsilon(1 + \varepsilon^{1+\varepsilon})\varphi$, where $\varepsilon < \pi/9$ and defined as the shocks strain (length-change over length). That is, $\varepsilon = W/(kr)$, where W is the gravity load on the hub, r is the ASW radius and k is the shock-spoke stiffness.

The ASW-to-wheel acceleration amplification factors can be read from Table 1 as $(\rho + \vartheta - \mu)/(\vartheta - \mu)$ for free rolling on slope, $(\rho + \varphi - \mu)/(\varphi - \mu)$ for towing on flat road, $(\rho + \varphi + \vartheta - \mu)/(\varphi + \vartheta - \mu)$ for towing downhill, and $(\rho + \varphi - \vartheta - \mu)/(\varphi - \vartheta - \mu)$ for towing uphill.

Accelerating force $F_{ACC} =$	Free rolling on slope	Towing on flat road	Towing downhill	Towing uphill	Driving equivalence
Wheel	$(\vartheta - \mu)W$	$(\varphi - \mu)W$	$(\varphi + \vartheta - \mu)W$	$(\varphi - \vartheta - \mu)W$	$F_{TOW} = T/r$
ASW	$(\rho + \vartheta - \mu)W$	$(\rho + \varphi - \mu)W$	$(\rho + \varphi + \vartheta - \mu)W$	$(\rho + \varphi - \vartheta - \mu)W$	$F_{TOW} = T/r^*$
Constraints	$\vartheta > 0, \varphi = 0$	$\vartheta = 0$	$\varphi > 0$	$\varphi > 0$	$T = \text{Torque}$
Definitions	$\mu = F_{FRI}/N$	$\mu = F_{FRI}/W$	$\vartheta > 0$	$\vartheta < 0$	$\varphi = F_{TOW}/W$

Table 1 Accelerating force comparison

Fuel Savings

Just by replacing a wheel with a commensurate ASW, using $\varepsilon_{MAX} = \pi/9$, we get up to 30% reduction in rolling resistance. Rolling resistance costs 1/8-1/2 engine power.

To save 1% fuel then, one need to reduce the rolling resistance by 2-8%. Considering that, a wheel-to-ASW swap may result in 4-15% fuel savings [2]. For illustration, see Table 2.

Vehicle types	Racing Cars	Cars and Vans	Highway Truck	Mining Trucks
Fraction of engine power consumed to overcome rolling resistance	1/8 Equal to 12.50%	1/5 Equal to 20.00%	1/3 Equal to 33.33%	1/2 Equal to 50.00%
That reduced by ASW	8.75%	14.00%	23.33%	35.00%

Maximum fuel savings by swapping wheels to ASWs	3.75% Rounded to 4%	6.00% Rounded to 6%	10.00% Rounded to 10%	15.00% Rounded to 15%
Average fuel savings by swapping wheels to ASWs	3.02% Rounded to 3%	4.83% Rounded to 5%	8.05% Rounded to 8%	12.08% Rounded to 12%
Vehicle types	Racing Cars	Cars and Vans	Highway Truck	Mining Trucks

Table 2. Fuel saving caps of various vehicle categories

Note that the mining truck category includes construction, mining and military equipments and vehicles. Note also that, just as tires are inflated to soft and hard, ASW shocks may hold low or high pressure, which affect k and thus ϵ , though less than compression ratio and fill pressure do. In any case, the average fuel savings is about $100|1-1/(1+\epsilon^{1+\epsilon})|=100|1-1/(1+\pi/9)|=20\%$ less than the maximum one (see Table 2). Note also that fuel savings is not the purpose but the benefit of the ASW. Also that, since two effects out of the three affecting ASW rolling resistance is temporary, actual fuel savings depend on driving conditions and thus may be reduced.

Finally note that while the shocks deflect greatly, the rim is not. That means that riding on ASWs is very comfortable even on very rough terrain. ASWs allowed to drive on hard pavement at elevated speed and extended time may need a rubber or polyurethane ribbed layer to protect the road from damage, especially under high wheel load. Military ASWs need such rubber rim cover.

Conclusion

Based on the physics of rolling, one may conclude that substituting a wheel with an ASW (Air Suspension Wheel), up to 30% rolling resistance reduction is achievable, which may translate to 4-15% maximum and 3-12% average fuel savings and emission and pollution reductions thereof. To derive this, we have revisited the physics of rolling and that of the wheel and ASW in free rolling on slope, in towing on flat road, as well as on slope downhill and uphill. We have proved that the cases of towing and driving with torque are equivalent and listed the wheel and ASW accelerating forces for the case of towing. The list gives multipliers over the wheel or ASW gravity load. The force amplification factor of the wheel-to-ASW switching, given here, incorporates the secondary effects of the elastic deformations over the loads. We have also listed the maximum and average fuel savings for various vehicle categories had these been replaced their wheels with commensurate ASWs. We have found that ASWs save four times more fuel in mining trucks than in racing cars. As much fuel is saved, that much emission is eliminated. ASWs ensure comfortable ride. Hard pavement are best protected if the ASW have a rubber shell over its rim.

References

- [1] http://en.wikipedia.org/wiki/Friction_coefficients
- [2] <http://www.michelintruck.com/michelintruck/tires-retreads/xone/xOne-fuel-savings.jsp#>
- [3] http://en.wikipedia.org/wiki/Rolling_resistance
- [4] Robert L. Peurifoy, Clifford J. Schexnayder, Aviad Shapira, Robert L. Schmitt: "Construction Planning, Equipment, and Methods", 8th Ed., University Textbook, ISBN978-0-07-340112-6, www.mhhe.com/peurifoy8e, Chapter 6, Mobile Equipment Power Requirement.

